Scheme of Study for BS Mathematics (Fall 2025)

Department of Mathematics, University of Poonch Rawalakot

Semester-I Scheme 2025			
Course Code	**Course Title**	**Credit Hours**	**Category**
MAT-3101	Calculus-I	3 (3-0)	Major
IDC-3102	Introduction to Programming	3 (2-1)	Interdisciplinary
GEN-3103	Quantitative Reasoning-I	3 (3-0)	General Education
GEN-3104	General Physics	3 (2-1)	General Education
GEN-3105	Functional English	3 (3-0)	General Education
GEN-3106	Applications of Information and Communication Technologies	3 (2-1)	General Education

Semester-III			
Course Code	Course Title	Credit hours	Category
GEN-4301	Applications Information and Communication Technologies	3(2-1)	General
GEN-4302	Quantitative Reasoning-II	3(3-0)	General
GEN-4303	Civics and Community Engagement	2(2-0)	General
MATH-4304	Calculus-III	3(3-0)	Major
MATH-4305	Numerical Analysis-I	3(3-0)	Major
PHY-4306	Waves, Oscillations and Thermodynamics	3(3-0)	Interdisciplinary
PHY-4307	Physics Lab-III	1(0-1)	Interdisciplinary

Total	***18(16-2)***	

Semester-V			
Course Code	Course Title	Credit hours	Category
MATH-5101	Real Analysis-I	3(3-0)	Major
MATH-5102	Differential Geometry	3(3-0)	Major
MATH-5103	Algebra-I	3(3-0)	Major
MATH-5104	Probability Theory	3(3-0)	Major
MATH-5105	Number Theory	3(3-0)	Major
Total		***15(15-0)***	

Semester-VII

Note: Any five courses from the following Elective Courses.

Course Code	Course Title	Credit hours
MAT-6301	Partial Differential Equations	3(3-0)
MAT-6302	Fluid Mechanics-I	3(3-0)
MAT-6303	Mathematical Statistics	3(3-0)
MAT-6304	Functional Analysis-II	3(3-0)
MAT-6305	Measure Theory	3(3-0)
MAT-6306	Modeling and Simulation	3(2-1)
MAT-6307	Optimization Theory	3(3-0)
MAT-6308	Numerical Methods for Partial-Differential Equations	3(3-0)
MAT-6309	Theory of Differential Equations	3(3-0)
MAT-6310	Analysis and Control of Nonlinear Systems	3(3-0)
MAT-6311	Computing Tools for Mathematicians	3(2-1)
MAT-6312	Vector and Tensor Analysis	3(3-0)
	Total	**15**

SEMESTER I

The first semester establishes the foundational knowledge base for the degree program, integrating core mathematics with essential skills in programming, quantitative reasoning, physical sciences, and communication.

1.1 MAT-3101: Calculus - I

Credit Hours: 3(3-0)
Category: Major

Course Description

This foundational course introduces the core concepts of single-variable calculus. It rigorously covers the principles of limits, continuity, and differentiation, culminating in an introduction to the concept of integration. The course is designed to develop both the computational skills and the theoretical understanding necessary for all subsequent studies in mathematics and related scientific fields.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Understand the concepts of limits, continuity, differentiation and integration.
- Apply differentiation and integration techniques to solve real-world problems.
- Analyze the behavior of functions using graphical and analytical methods.

Comprehensive Course Contents

- **Week 1-2: Pre-Calculus Review.** Functions and their graphs, domain and range. Operations on functions. Trigonometric, exponential, and logarithmic functions.
- **Week 3-4: Limits and Continuity.** The concept of a limit, calculating limits using limit laws, the Squeeze Theorem. Continuity, types of discontinuities, the Intermediate Value Theorem.
- **Week 5-6: The Derivative.** The derivative as the slope of a tangent line and as a rate of change. The limit definition of the derivative. Differentiability and continuity.
- **Week 7-8: Differentiation Rules.** The power, product, and quotient rules. Derivatives of trigonometric functions. The chain rule.
- **Week 9-10: Advanced Differentiation.** Implicit differentiation. Derivatives of exponential and logarithmic functions. Derivatives of inverse trigonometric functions.
- **Week 11: Midterm Examination.**
- **Week 12-13: Applications of Differentiation.** Related rates. Finding absolute and local extrema. The Mean Value Theorem and Rolle's Theorem.
- **Week 14-15: Further Applications.** The first and second derivative tests for curve sketching (concavity, points of inflection). L'Hôpital's Rule. Optimization problems.

- **Week 16: Antiderivatives and Introduction to Integration.** Antiderivatives, indefinite integrals. Introduction to the definite integral and the Fundamental Theorem of Calculus.

Recommended Books

- **Primary:** Stewart, J. (2021). *Calculus: Early Transcendentals* (9th ed.). Cengage Learning.
- **Supplementary:** Strang, G. (2017). *Calculus* (3rd ed.). Wellesley-Cambridge Press.
- **Supplementary:** Thomas, G. B., Weir, M. D., & Hass, J. R. (2018). *Thomas' Calculus* (14th ed.). Pearson.

1.2 IDC-3102: Introduction to Programming

Credit Hours: 3(2-1)
Category: Interdisciplinary

Course Description

This course is designed for students with no prior programming experience. It introduces the fundamental principles of computer programming and problem-solving using a high-level language, such as Python. Core topics include variables, data types, control structures (conditionals and loops), functions, and an introduction to basic data structures and object-oriented concepts. A significant lab component provides hands-on practice.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Understand the fundamental concepts of programming (variables, loops, and conditionals).
- Write basic programs to solve problems using structured programming techniques.
- Debug and test programs for accuracy and efficiency.

Comprehensive Course Contents

- **Week 1-2: Introduction to Computing.** Algorithms, flowcharts, pseudo-code. Introduction to the programming environment. Basic syntax, "Hello World" program.
- **Week 3-4: Variables and Data Types.** Primitive data types (integers, floats, strings, booleans). Variable declaration and assignment. Basic input/output operations.
- **Week 5-6: Operators and Expressions.** Arithmetic, relational, and logical operators. Operator precedence. Type casting.
- **Week 7**
- **Week 8-9: Control Structures I.** Conditional statements: if, if-else, if-elif-else. Nested conditionals.
- **Week 10-11: Control Structures II.** Iteration and loops: for loops, while loops. Loop control statements (break, continue).

- **Week 12-13: Functions.** Defining and calling functions. Parameters and return values. Scope of variables (local and global).
- **Week 14: Data Structures.** Introduction to lists (arrays) and strings. Basic operations: indexing, slicing, iteration.
- **Week 15: File Handling and Exception Handling.** Reading from and writing to text files. Introduction to error handling with try-except blocks.
- **Week 16: Introduction to Object-Oriented Programming.** Concepts of classes and objects. Final project development and submission.

Recommended Books

- **Primary:** Matthes, E. (2019). *Python Crash Course* (2nd ed.). No Starch Press.
- **Supplementary:** Sweigart, A. (2019). *Automate the Boring Stuff with Python* (2nd ed.). No Starch Press.
- **Supplementary:** Downey, A. B. (2015). *Think Python: How to Think Like a Computer Scientist* (2nd ed.). O'Reilly Media.

1.3 GEN-3103: Quantitative Reasoning-I

Credit	**Hours:**	3(3-0)
Category:	General	Education

Prerequisites: Strong foundation in Intermediate Mathematics

Course Description

This foundational course bridges secondary school mathematics with university-level mathematical thinking. It develops rigorous reasoning skills through the study of formal logic, set theory, proof techniques, and fundamental mathematical structures. The course emphasizes the transition from computational mathematics to abstract mathematical thinking, preparing students for advanced mathematics courses while complementing the concurrent Calculus I course.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Construct and analyze mathematical arguments using formal logic and proof techniques
- Apply set theory and function concepts to mathematical problem-solving
- Demonstrate understanding of number systems and their algebraic properties
- Solve problems using combinatorial reasoning and basic probability theory
- Communicate mathematical ideas with precision and clarity
- Translate between mathematical formalism and intuitive understanding

Comprehensive Course Contents

- **Week 1-3: Mathematical Logic and Set Theory**
    - Propositional logic: logical connectives, truth tables, logical equivalence
    - Predicate logic: quantifiers, nested quantifiers
    - Basic set operations: union, intersection, complement, Cartesian product
    - Set identities and proofs of set equality
    - Introduction to mathematical proofs: direct proof, proof by contraposition
- **Week 4-6: Proof Techniques and Number Systems**
    - Proof by contradiction and proof by cases
    - Mathematical induction and strong induction
    - Properties of integers: divisibility, prime factorization, Euclidean algorithm
    - Modular arithmetic and congruence relations
    - Construction and properties of rational numbers

Week 7: Midterm

- **Week 8-9: Functions and Relations**
    - Functions: injective, surjective, bijective properties
    - Function composition and inversion
    - Relations: equivalence relations, partial orders
    - Cardinality: finite sets and introduction to infinite sets
- **Week 10-12: Combinatorics and Discrete Probability**
    - Basic counting principles: product rule, sum rule, inclusion-exclusion
    - Permutations, combinations, and binomial coefficients
    - Pigeonhole principle and applications
    - Sample spaces, events, and basic probability axioms
    - Conditional probability and independence
- **Week 13-15: Introduction to Mathematical Modeling**
    - Modeling with sequences and recurrence relations
    - Introduction to graph theory: basic definitions and examples
    - Elementary number theory applications
    - Problem-solving strategies and mathematical communication

Recommended Books

- **Primary:** Hammack, R. H. (2018). *Book of Proof* (3rd ed.). Chapman and Hall/CRC.
- **Primary:** Rosen, K. H. (2019). *Discrete Mathematics and Its Applications* (8th ed.). McGraw-Hill.
- **Supplementary:** Velleman, D. J. (2019). *How to Prove It: A Structured Approach* (3rd ed.). Cambridge University Press.
- **Reference:** Chartrand, G., Polimeni, A. D., & Zhang, P. (2018). *Mathematical Proofs: A Transition to Advanced Mathematics* (4th ed.). Pearson.

1.4 GEN-3104: General Physics

Credit Hours: 3(2-1)
Category: General Education

Course Description

This is an algebra-based introductory course in physics that covers the fundamental principles of mechanics. Topics include kinematics in one and two dimensions, Newton's laws of motion, work, energy, power, linear momentum, and collisions. The course emphasizes conceptual understanding and problem-solving skills, supported by a laboratory component for hands-on exploration of physical laws.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Understand the basic concepts such as mechanics, and electromagnetism.
- Apply physical principles to solve problems in mechanics and waves.
- Analyze experimental data to validate physical laws and theories.

Comprehensive Course Contents

- **Week 1-2: Introduction.** Physics and measurement, standards of length, mass, and time. Dimensional analysis. Introduction to vectors.
- **Week 3-4: Motion in One Dimension.** Position, velocity, and speed. Instantaneous velocity and speed. Acceleration. Motion with constant acceleration. Freely falling objects.
- **Week 5-6: Motion in Two Dimensions.** Position, velocity, and acceleration vectors. Projectile motion. Uniform circular motion.
- **Week 7: Midterm**
- **Week 8-9: The Laws of Motion.** Newton's First, Second, and Third Laws. The gravitational force and weight. Forces of friction.
- **Week 9-10: Applications of Newton's Laws.** Objects in equilibrium. Objects experiencing a net force. Uniform circular motion and its applications.
- **Week 11-12: Energy and Energy Transfer.** Work done by a constant force. The scalar product of two vectors. Work done by a varying force. Kinetic energy and the work-kinetic energy theorem. Potential energy. Conservative and non-conservative forces.
- **Week 13-14: Conservation of Energy.** Conservation of mechanical energy. Relationship between conservative forces and potential energy. Conservation of total energy. Power.
- **Week 15-16: Linear Momentum and Collisions.** Linear momentum and its conservation. Impulse and momentum. Collisions in one and two dimensions (elastic and inelastic).

Recommended Books

- **Primary:** Young, H. D., & Freedman, R. A. (2019). *University Physics with Modern Physics* (15th ed.). Pearson.

- **Supplementary:** Halliday, D., Resnick, R., & Walker, J. (2020). *Fundamentals of Physics* (11th ed.). Wiley.
- **Supplementary:** Serway, R. A., & Jewett, J. W. (2018). *Physics for Scientists and Engineers* (10th ed.). Cengage Learning.

List of Experiments:

1. To learn the use of the vernier caliper, screw gauge, and to find the diameter, thickness, and radius of curvature of given objects.
2. To determine the acceleration due to gravity (g) by using a simple pendulum.
3. To study the relationship between force, mass, and acceleration using Atwood's machine (Verification of Newton's Second Law).
4. To find the moment of inertia of a flywheel.
5. To study simple harmonic motion (SHM) using a spring–mass system and determine the spring constant.
6. To study the motion of a body rolling down an inclined plane and compare experimental acceleration with theoretical values.
7. To determine the work done by a variable force using a force-displacement graph.
8. To determine the modulus of rigidity of a wire using a torsional pendulum.
9. To determine the Young's modulus of a wire using Searle's apparatus.
10. To verify Hooke's law and determine the force constant of a spring.

1.5 GEN-3105: Functional English

Credit Hours: 3(3-0)
Category: General Education

Course Description

This course is designed to enhance students' English language skills for academic and professional communication. It focuses on developing proficiency in grammar, vocabulary, reading comprehension, and basic writing. The course aims to build confidence and competence in using English effectively in diverse real-world scenarios.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Apply enhanced English communication skills through effective use of word choices, grammar, and sentence structure.
- Comprehend a variety of literary/non-literary written and spoken texts in English.
- Communicate effectively in diverse contexts with cultural awareness.

Comprehensive Course Contents

Grammar:

- Basics of grammar
- Parts of speech and their use in communication
- Sentence structure
- Correct use of Tenses
- Active and passive voice
- Practice in unified sentences (unity and coherence)
- Analysis of Phrase, Clause and sentence structures
- Transitive and Intransitive Verbs
- Punctuation and Spellings

Reading skills:

- Comprehension skills
- Literal understanding of text, reading between lines (interpret text), reading beyond lines (to assimilate, integrate knowledge)
- Answers to the questions on a given text

Discussion:

- General topics and everyday conversation (topics for discussion to be at the discretion of the teacher keeping in view the level of the students)
- Introducing ourselves, describing things, recounting past events, agreeing and disagreeing, compare and contrast

Listening:

- To be improved by showing documentaries/ films carefully selected by subject teacher
- Listening and note taking

Translation Skills:

- Urdu to English

Writing Skills:

- Paragraph Writing

  Basic structure of paragraph and guidelines for writing an effective paragraph

Speaking Skills:

- Presentation Skills
- Introduction (types of presentation, structure of presentation)
- Prepared and unprepared talks

Note: Extensive reading is required for vocabulary building

Recommended Books:

1. **Functional English**

a) Grammar

1. Practical English Grammar by A. J. Thomson and A. V. Martinet. Exercises 1. Third edition. Oxford University Press. 1997. ISBN 0194313492
2. Practical English Grammar by A. J. Thomson and A. V. Martinet. Exercises 2. Third edition. Oxford University Press. 1997. ISBN 0194313506

b) Writing

1. Writing. Intermediate by Marie-Christine Boutin, Suzanne Brinand and Francoise Grellet. Oxford Supplementary Skills. Fourth Impression 1993. ISBN 0 19 435405 7 Pages 20-27 and 35-41.

c) Reading/Comprehension

1. Reading. Upper Intermediate. Brain Tomlinson and Rod Ellis. Oxford Supplementary Skills. Third Impression 1992. ISBN 0 19 453402 2.

d) Speaking

1) Ellen, K. 2002. Maximize Your Presentation Skills: How to Speak, Look and Act on Your Way to the Top
2) Hargie, O. (ed.) Hand book of Communications Skills
3) Mandel, S. 2000. Effective Presentation Skills: A Practical Guide Better Speaking
4) Mark, P. 1996. Presenting in English. Language Teaching Publications

- **Primary:** Eastwood, J. (2002). *Oxford Practice Grammar*. Oxford University Press.
- **Supplementary:** Thomson, A. J., & Martinet, A. V. (1986). *A Practical English Grammar*. Oxford University Press.
- **Supplementary:** Raman, M., & Sharma, S. (2011). *Technical Communication: Principles and Practice*. Oxford University Press.

1.6 GEN-3106: Applications of Information & Communication Technologies

Credit Hours: 3(2-1)
Category: General Education

Course Description

This course provides a practical, hands-on exploration of Information and Communication Technologies (ICT) and software tools. Students will gain proficiency in essential productivity applications (word processing, spreadsheets, presentations), online communication and collaboration tools, and cloud-based storage. The course also covers the principles of digital citizenship, including online ethics, security, and information literacy.

Course Learning Outcomes

Upon successful completion of this course, students will be able to:

- Explain the fundamental concepts, components, and scope of Information and Communication Technologies (ICT).
- Apply ICT platforms and tools to address basic needs in daily, academic, and professional life.
- Understand the ethical and legal considerations in the use of ICT platforms and tools.

Comprehensive Course Contents

Objective

1. Explain the fundamental concepts, components, and scope of Information and Communication Technologies (ICT).
2. Identify uses of various ICT platforms and tools for different purposes.

Course Contents:

1. Introduction to Information and Communication Technologies:

- Components of Information and Communication Technologies (basics of hardware, software, ICT platforms, networks, local and cloud data storage etc.).

- Scope of Information and Communication Technologies (use of ICT ineducation, business, governance, healthcare, digital media and entertainment, etc.).
- Emerging technologies and future trends.

2. Basic ICT Productivity Tools:

- Effective use of popular search engines (e.g., **Google,** Bing, etc.) to explore World Wide Web.
- Formal communication tools and etiquettes (Gmail, Microsoft Outlook, etc.).
- Microsoft Office Suites (Word, Excel, PowerPoint).
- Google Workspace (Google Does, Sheets, Slides).
- Dropbox (Cloud storage and file sharing), Google Drive (Cloud storage with Google Docs integration) and Microsoft OneDrive (Cloud storage with Microsoft Office integration).
- Evernote (Note-taking and organization applications) and OneNote (Microsoft's digital notebook for capturing and organizing ideas).

  Video conferencing (Google Meet, Microsoft Teams, Zoom, etc.).
- Social media applications (LinkedIn, Facebook, Instagram, etc .).

3. ICT in Education:

- Working with learning management systems (Moodie, Canvas, Google Classrooms, etc.).
- Sources of online education courses (Coursera, edX , Udemy, Khan Academy, etc.).
- Interactive multimedia and virtual classrooms.

4. ICT in Health and Well-being:

- Health and fitness tracking devices and applications (Google Fit, Samsung Health, Apple Health, Xiaomi Mi Band, Runkeeper, etc.).
- Telemedicine and online health consultations (OLADOC, Sehat Kahani,
  Marham, etc.).

5. ICT in Personal Finance and Shopping:

- Online banking and financial management tools (JazzCash, Easypaisa, Zong PayMax, I LINK and MNET, Keenu Wallet, etc.).
- E-commerce platforms (Daraz.pk, Telemart, Shophive, etc.)

6. Digital Citizenship and Online Etiquette:

- Digital identity and online reputation.
- Netiquette and respectful online communication.
- Cyberbullying and online harassment.

7. Ethical Considerations in Use of ICT Platforms and Tools:

- Intellectual property and copyright issues.
- Ensuring originality in content creation by avoiding plagiarism and unauthorized use of information sources.
- Content accuracy and integrity (ensuring that the content shared through ICT platforms is free from misinformation, fake news, and manipulation).

Practical Requirement

As part of the overall learning requirements, the course will include:

1. Guided tutorials and exercises to ensure that students are proficient in commonly used software applications such as word processing software (e.g., Microsoft Word), presentation software (e.g., Microsoft PowerPoint), spreadsheet software (e.g., Microsoft Excel) among such other tools. Students may be assigned practical tasks that require them to create documents, presentations, and spreadsheets etc.
2. Assigning of tasks that involve creating, managing, and organizing files and folders on both local and cloud storage systems. Students will practice file naming conventions, creating directories, and using cloud storage solutions (e.g., Google Drive, OneDrive).
3. The use of online learning management systems (LMS) where students can access course materials, submit assignments, participate in discussion forums, and take quizzes or tests. This will provide students with the practical experience with online platforms commonly used in education and the workplace.

Reading Materials

I."Discovering Computers" by Vermaat, Shaffer, and Freund.
2."GO! with Microsoft Office" Series by Gaskin, Vargas, and McLellan.
3. "Exploring Microsoft Office" Series by Grauer and Poatsy.
4. "Computing Essentials" by Morley and Parker.
5. "Technology in Action" by Evans, Martin, and Poatsy.

Semester-III

GEN-4301: Applications of Information and Communication Technologies 3 (2-1)

Objective

2. Explain the fundamental concepts, components, and scope of Information and Communication Technologies (ICT).

2. Identify uses of various ICT platforms and tools for different purposes.

Course Contents:

1. Introduction to Information and Communication Technologies:

- Components of Information and Communication Technologies (basics of hardware, software, ICT platforms, networks, local and cloud data storage etc.).

- Scope of Information and Communication Technologies (use of ICT ineducation, business, governance, healthcare, digital media and entertainment, etc.).
- Emerging technologies and future trends.

2. Basic ICT Productivity Tools:

- Effective use of popular search engines (e.g., **Google,** Bing, etc.) to explore World Wide Web.
- Formal communication tools and etiquettes (Gmail, Microsoft Outlook, etc.).
- Microsoft Office Suites (Word, Excel, PowerPoint).
- Google Workspace (Google Does, Sheets, Slides).
- Dropbox (Cloud storage and file sharing), Google Drive (Cloud storage with Google Docs integration) and Microsoft OneDrive (Cloud storage with Microsoft Office integration).
- Evernote (Note-taking and organization applications) and OneNote (Microsoft's digital notebook for capturing and organizing ideas).

  Video conferencing (Google Meet, Microsoft Teams, Zoom, etc.).
- Social media applications (LinkedIn, Facebook, Instagram, etc .).

3. ICT in Education:

- Working with learning management systems (Moodie, Canvas, Google Classrooms, etc.).
- Sources of online education courses (Coursera, edX , Udemy, Khan Academy, etc.).
- Interactive multimedia and virtual classrooms.

4. ICT in Health and Well-being:

- Health and fitness tracking devices and applications (Google Fit, Samsung Health, Apple Health, Xiaomi Mi Band, Runkeeper, etc.).
- Telemedicine and online health consultations (OLADOC, Sehat Kahani,
  Marham, etc.).

5. ICT in Personal Finance and Shopping:

- Online banking and financial management tools (JazzCash, Easypaisa, Zong PayMax, I LINK and MNET, Keenu Wallet, etc.).
- E-commerce platforms (Daraz.pk, Telemart, Shophive, etc.)

6. Digital Citizenship and Online Etiquette:

- Digital identity and online reputation.
- Netiquette and respectful online communication.
- Cyberbullying and online harassment.

7. Ethical Considerations in Use of ICT Platforms and Tools:

- Intellectual property and copyright issues.
- Ensuring originality in content creation by avoiding plagiarism and unauthorized use of information sources.
- Content accuracy and integrity (ensuring that the content shared through ICT platforms is free from misinformation, fake news, and manipulation).

Practical Requirement

As part of the overall learning requirements, the course will include:

2. Guided tutorials and exercises to ensure that students are proficient in commonly used software applications such as word processing software (e.g., Microsoft Word), presentation software (e.g., Microsoft PowerPoint), spreadsheet software (e.g., Microsoft Excel) among such other tools. Students may be assigned practical tasks that require them to create documents, presentations, and spreadsheets etc.
2. Assigning of tasks that involve creating, managing, and organizing files and folders on both local and cloud storage systems. Students will practice file naming conventions, creating directories, and using cloud storage solutions (e.g., Google Drive, OneDrive).
3. The use of online learning management systems (LMS) where students can access course materials, submit assignments, participate in discussion forums, and take quizzes or tests. This will provide students with the practical experience with online platforms commonly used in education and the workplace.

Reading Materials

I."Discovering Computers" by Vermaat, Shaffer, and Freund.
2."GO! with Microsoft Office" Series by Gaskin, Vargas, and McLellan.
3. "Exploring Microsoft Office" Series by Grauer and Poatsy.
4. "Computing Essentials" by Morley and Parker.
5. "Technology in Action" by Evans, Martin, and Poatsy.

GEN: 4302 Quntitative Reasoning –II Credit Hrs. 3(3-0)

Course Objectives:

Summarizing, interpreting, and presenting quantitative data in mathematical forms, such as graphs, charts, tables, or mathematical text. Construct or compute representations of data using mathematical forms or equations as models and use statistical methods to assess their accuracy.

Course Contents:

Logic, Logical and Critical Reasoning: Introduction and importance of logic; Inductive, deductive and abductive approaches of reasoning; Propositions, arguments (valid; invalid), logical connectives, truth tables and propositional equivalences; Logical fallacies; Venn Diagrams; Predicates and quantifiers; Quantitative reasoning exercises using logical reasoning concepts and techniques. **Mathematical Modeling and Analyses:** Introduction to deterministic models; Use of linear functions for modeling in real-world situations; Modeling with the system of linear equations and their solutions; Elementary introduction to derivatives in mathematical modeling; Linear and exponential growth and decay models; Quantitative reasoning exercises using mathematical modeling. **Statistical Modeling and Analyses**: Introduction to probabilistic models; Bivariate analysis, scatter plots; Simple linear regression model and correlation analysis; Basics of estimation and confidence interval; Testing of hypothesis (z-test; t-test); Statistical inference in decision making; Quantitative reasoning exercises using statistical modeling.

Recommended Books:

1. Using and Understanding Mathematics: A Quantitative Reasoning Approach by Bennett, J. O., Briggs, W. L., & Badalamenti, A.
2. Discrete Mathematics and its Applications byy Kenneth H. Rosen.
3. Discrete Mathematics with Applications by Susanna S. Epp.
4. Applied Mathematics for Business, Economics and Social Sciences by Frank S. Budnick.
5. Elementary Statistics: A Step-by-Step Approach by Allan Bluman.
6. Introductory Statistics by Prem S. Mann.
7. Applied Statistical Modeling by Salvatore Babones.
8. Barrons SAT by Sharvon Weiner Green, M.A and Ira K. Wolf.

GEN-4303 Civics and Community Engagement 2 (2-0)

Learning Outcome

By the end of this course, students will be able to:

1. Demonstrate fundamental understanding of civics, government, citizenship and civil society.
2. Understand the concept of community and recognize the significance of community engagement for individuals and groups.
3. Recognize the importance of diversity and inclusivity for societal harmony and peaceful co- existence.

Course Contents:

1. **Civics and Citizenship:**
   - Concepts of civics, citizenship, and civic engagement.
   - Foundations of modern society and citizenship.
   - Types of citizenship: active, participatory, digital, etc.
2. **State, Government and Civil Society:**
   - Structure and functions of government in Pakistan.
   - The relationship between democracy and civil society.
   - Right to vote and importance of political participation and representation.
3. **Rights and Responsibilities:**
   - Overview of fundamental rights and liberties of citizens under Constitution of Pakistan 1973.
   - Civic responsibilities and duties.
   - Ethical considerations m c1v1c engagement (accountability, non-violence, peaceful dialogue, civility, etc.)
4. **Community Engagement:**
   - Concept, nature and characteristics of community.
   - Community development and social cohesion.
   - Approaches to effective community engagement.
   - Case studies of successful community driven initiatives.
5. **Advocacy and Activism:**
   - Public discourse and public opinion.
   - Role of advocacy in addressing social issues.
   - Social action movements.
6. **Digital Citizenship and Technology:**
   - The use of digital platforms for civic engagement.
   - Cyber ethics and responsible use of social media.
   - Digital divides and disparities (access, usage, socioeconomic, geographic, etc.) and their impacts on citizenship.

7. **Diversity, Inclusion and Social Justice:**
    - Understanding diversity in society (ethnic, cultural, economic, political etc.).
    - Youth, women and minorities' engagement in social development.
    - Addressing social inequalities and injustices in Pakistan.
    - Promoting inclusive citizenship and equal rights for societal harmony and peaceful co- existence.

Suggested Reading

1. "Civics Today: Citizenship, Economics, & You" by McGraw-Hill Education.
2. "Citizenship in Diverse Societies" by Will Kymlicka and Wayne Norman.
3. "Engaging Youth in Civic Life" by James Youniss and Peter Levine.
4. "Digital Citizenship in Action: Empowering Students to Engage in Online Communities" by Kristen Mattson.
5. "Globalization and Citizenship: In the Pursuit of a Cosmopolitan Education" by Graham Pike and David Selby.

"Community Engagement: Principles, Strategies, and Practices" by Becky J. Feldpausch and Susan M. Omilian.

MATH-4304 Calculus-III Credit Hrs. 3(3-0)

Course Objectives:

This course extends the study of infinite series, conic sections, and calculus to multivariable functions, including partial differentiation, multiple integration, and vector calculus.

Course Contents:

Sequences, Monotone Sequences, Infinite Series, Convergence Tests, The Comparison, Ratio, and Root Tests, Alternating Series, Absolute and Conditional Convergence, Maclaurin and Taylor Polynomials, Maclaurin and Taylor Series; Power Series, Convergence of Taylor Series, Differentiating and Integrating Power Series, Modeling with Taylor Series, Parametric Equations, Tangent Lines and Arc Length for Parametric Curves, Polar Coordinates, Tangent Lines, Arc Length, and Area for Polar Curves, Conic Sections, Rotation of Axes, Second-Degree Equations, Conic Sections in Polar Coordinates, Rectangular Coordinates in 3-Space, Spheres, Cylindrical Surfaces, Parametric Equations of Lines, Planes in 3-Space, Quadric Surfaces, Functions of Two or More Variables, Limits and Continuity, Partial Derivatives, Differentiability, Differentials, and Local Linearity, The Chain Rule, Directional Derivatives and Gradients, Tangent Planes and Normal Vectors, Maxima and Minima of Functions of Two Variables, Lagrange

Multipliers, Cylindrical and Spherical Coordinates, Double Integrals, Double Integrals over Nonrectangular Regions, Double Integrals in Polar Coordinates, Surface Area, Parametric Surfaces, Triple Integrals, Change of Variables in Multiple Integrals, Jacobians.

Recommended books:

1. Multivariable Calculus" by James Stewart
2. Calculus Volume 3" by Edwin Herman and Gilbert Strang
3. Calculus: Early Transcendental" by Howard Anton, Irl Bivens and Stephen Davis

MATH-4305 Numerical Analysis-I Credit Hrs. 3(3-0)

Course Objectives:

This course introduces students to fundamental numerical methods for solving mathematical problems and approximating solutions to mathematical equations and functions.

Course Contents:

Introduction to Numerical Analysis: Overview of numerical methods and their significance, Sources of error in numerical computations, Root Finding and Nonlinear Equations**:** Bisection method**,** False position method**,** Newton-Raphson method**,** Secant method**,** Fixed point iteration method**,** Convergence and stopping criteria**,** Interpolation and Polynomial Approximation: Lagrange interpolation**,** Newton's divided-difference interpolation**,** Hermit interpolation**,** Polynomial interpolation error**,** Numerical Differentiation, and Integration: Forward, backward, and central difference approximations**,** Numerical integration: trapezoidal rule, Simpson's rule**,** Error analysis for differentiation and integration**,** Systems of Linear Equations: Gaussian elimination methods**, Gauss Jordan method, matrix inversion,** LU decomposition**,** Iterative methods: Jacobi, Gauss-Seidel, SOR.

Recommended books:

1. Numerical Analysis" by Richard L. Burden and J. Douglas Faires
2. An Introduction to Numerical Analysis" by Endre Süli and David F. Mayers
3. Introduction to Numerical Analysis" by J. Stoer and R. Bulirsch
4. Numerical Methods for Scientists and Engineers" by R.W. Hamming

PHY-4306 Waves, Oscillations, and Thermodynamics Hrs. 3(3-0)

Course Objective:

1. To understand the basics of waves, the mechanism of wave production, propagation and interaction with other waves.
2. Use of the basic concept of waves in their application in daily life.

Course Contents:

Harmonic Oscillations:

Simple harmonic motion (SHM), Obtaining and solving the basic equations of motion x (t), v (t), a (t), Longitudinal and transverse Oscillations, and Energy considerations in SHM. Application of SHM, Torsional oscillator, Physical pendulum, simple pendulum, SHM,, and uniform circular motion, Combinations of harmonic motions, Lissajous patterns, Damped harmonic motion, Equation of damped harmonic motion, Quality factor, discussion of its solution, Forced oscillations and resonances, Equation of forced oscillation, Discussion of its solution, Natural frequency, Resonance, Examples of resonance.

Waves in Physical Media:

Mechanical waves, Travelling waves, Phase velocity of traveling waves, Sinusoidal waves, Group speed and dispersion, Waves speed, Mechanical analysis, Wave equation, Discussion of solution, Power, and intensity in wave motion, Derivation & discussion, Principle of superposition (basic ideas), Interference of waves, Standing waves. Phase changes on reflection, Beats Phenomenon, Analytical treatment, Electromagnetic Spectrum, Doppler Effect Reflection and Refraction of light waves, Total internal reflection, Plane and Spherical Mirrors, Double slit Interference, Michelson and Fabry-Perot Interferometers.

Thermodynamics:

Review of previous concepts. The first law of thermodynamics and its applications to adiabatic, isothermal, cyclic, and free expansion, Reversible and irreversible processes, the Second Law of thermodynamics, Carnot theorem, Carnot engines. Heat engine, Refrigerators, Calculation of efficiency of heat engines. Thermodynamic temperature scale: Absolute zero: Entropy, Entropy in a reversible process, Entropy in an irreversible process. Entropy & second law. Entropy & probability.

Recommended Books:

1. Halliday, D. Resnick, Krane, Physics, Vol. I & II, John Wiley, 5th ed. 1999.
2. N.K. Bajaj, The Physics of Waves & Oscillations, Tata McGraw-Hill Publishing company Limited, 1986.
3. H. J. Pain, The Physics of Vibrations and Waves, 5th Edition 1999.

4. J. A. Edminister Schaum's Outline Series; Theory and Problems of Electromagnetism, McGraw-Hill Book Co., 1986.
5. J. F. Lee and F. W. Sears, Thermodynamics, Addison-Wesley 1954.
6. A. J. Pointon, Introduction to Statistical Physics, Longman 1967.
7. M. W. Zemansky, Heat and Thermodynamics, 3rd Edition, McGraw Hill, 1951.
8. Reif, Statistical Physics, Berkley Physics series, McGraw Hill 1965.
10. M. M. Abbott, Schaum's Outline of Thermodynamics, McGraw-Hill Professional Book Group, 1995.

PHY-4307 Physics Lab-III Credit Hrs. 1(0-1)

Course Objectives:

To know the electrical circuit elements and their experimental measurement to give an understanding of electrical circuits and the use of CRO.

List of Experiments:

1. 1 Calibration of an Ammeter and a Voltmeter by potentiometer
2. Comparison of capacities by ballistic galvanometer.

List of Experiments:

1. 1 Calibration of an Ammeter and a Voltmeter by potentiometer
2. Comparison of capacities by ballistic galvanometer.
3. Measurement of self/mutual inductance.
4. To convert a Western-type galvanometer into an ammeter reading up to 1 ampere (0-1amp range)
5. To convert a moving coil (Western type) galvanometer into a voltmeter reading up to 3 volts (0-3Volt range)
6. Setup of a RLC series circuit. Draw its frequency response curve and find the values of resonance frequency bandwidth and quality factor.
7. Setup of a RLC parallel circuit. Draw its frequency response curve and find the values of resonance frequency bandwidth and quality factor.
8. To determine thermal Emf and plot temperature diagram.
9. Calibration of the thermocouple by the potentiometer.
10. To study the network theorems (Superposition, Thevinin, Norton).

Recommended Books:

1. G L Squires, Practical Physics, 3rd Edition, Cambridge University Press

2. Nolan and Bigliani, Experiments in Physics, Surjeet Pub Ind.

Semester-V

MATH-5101: Real Analysis-I Credit Hrs. 3(3-0)

Course Contents:

The Real and Complex, Number Systems: Introduction, Ordered Sets, Fields, The Real Field, The Extended Real Number System, The Complex Field, Euclidean Spaces. Basic Topology: Finite, Countable, and Uncountable Sets, Metric Spaces, Compact Sets, Perfect Sets, Connected Sets. Numerical Sequences and Series: Convergent Sequences, Subsequences, Cauchy Sequences, Upper and Lower Limits, Some Special Sequences, Series, Series of Nonnegative Terms, The Number *e*, The Root and Ratio Tests, Power Series, Summation by Parts, Absolute Convergence, Addition and Multiplication of Series, Rearrangements. Continuity: Limits of Functions, Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite Limits and Limits at Infinity. Differentiation: The Derivative of a Real Function, Mean Value Theorems, The Continuity of Derivatives, L'Hospital's Rule, Derivatives of Higher Order, Taylor's Theorem, Differentiation of Vector-valued Functions.

Recommended Textbooks:

[1] "Principles of Mathematical Analysis" by Walter Rudin
[2] “Understanding Analysis” by Stephen Abbott
[3] “Analysis Volume I and II” by Terrence Tao
[4] “The Way of Analysis” by Robert S. Strichartz
[5] “Introduction to Real Analysis” by Robert G. Bartle and Donald R. Sherbert.

MATH-5102: Differential Geometry Credit Hrs. 3(3-0)

Course Description: This course introduces students to the basic concepts of differential geometry in Euclidean space, focusing on curves and surfaces.

Course Contents:

1. **Introduction to Differential Geometry**
    - Motivation and applications of differential geometry
    - Tangent vectors, velocity, and acceleration
2. **Curves in Euclidean Space**
    - Parametric curves and arc length
    - Tangent vectors, curvature, and torsion
    - Frenet-Serret formulas and osculating plane
3. **Surfaces in Euclidean Space**
    - Parametric surfaces and their patches
    - Tangent plane, normal vector, and orientation
    - First and second fundamental forms

4. **Gaussian and Mean Curvature**
    - Gaussian curvature and principal curvatures
    - Mean curvature and minimal surfaces
    - Applications in geometry and physics
5. **Curvature and Torsion of Curves**
    - Curvature and torsion formulas for space curves
    - Curvature and torsion of plane curves
    - Curvature properties and applications

Recommended Textbooks:
[1] "Elementary Differential Geometry" by Andrew Pressley
[2] "Differential Geometry of Curves and Surfaces" by Manfredo P. do Carmo

MATH-5103: Algebra-I Credit Hrs. 3(3-0)

Course Description: This course provides an introduction to the basic concepts and structures of abstract algebra, including groups, rings, and fields. Topics covered include group theory, subgroups, normal subgroups, quotient groups, homomorphisms, isomorphisms, and group actions.

Course Contents:

Groups: Introduction to Groups, Basic Axioms and Examples, Dihedral Groups, Symmetric Groups, Matrix Groups, The Quaternion Group, Homomorphisms and Isomorphisms, Group Actions. **Subgroups:** Definition and Examples, Centralizers and Normalizers, Stabilizers and Kernels, Cyclic Groups and Cyclic Subgroups, Subgroups Generated by Subsets of a Group, The Lattice of Subgroups of a Group. **Quotient Groups and Homomorphisms:** Definitions and Examples, More on Cosets and Lagrange's Theorem, The Isomorphism Theorems, Composition Series and the Holder Program, Transpositions and the Alternating Group. **Group Actions:** Group Actions and Permutation Representations, Groups Acting on Themselves by Left Multiplication-Cayley's Theorem, Groups Acting on Themselves by Conjugation-The Class Equation, Automorphisms, The Sylow Theorems, The Simplicity of A_n. **Direct and Semidirect**

Products and Abelian Groups: Direct Products, The Fundamental Theorem of Finitely Generated Abelian Groups, Table of Groups of Small Order, Recognizing Direct Products, Semidirect Products. **Further Topics in Group Theory:** p-groups, Nilpotent Groups, and Solvable Groups.

Recommended Textbooks:

[1] "A First Course in Abstract Algebra" by John B. Fraleigh
[2] “Abstract Algebra” by David S. Dummit and Richard M. Foote
[3] “Topics in Algebra” by I.N. Herstein
[4] “A Course in Abstract Algebra” by Vijay K. Khanna and S. K. Bhambri

MATH-5104: Probability Theory Credit Hrs. 3(3-0)

Course Contents:

Introduction to Probability: Basic concepts and terminology; Axioms and rules of probability; Counting techniques and combinatorics; Sample spaces and events; Venn diagrams and set operations. Discrete Random Variables: Definition and examples of discrete random variables; Probability mass function and cumulative distribution function; Bernoulli, binomial, geometric, negative binomial, hypergeometric, Poisson distributions; Expectation, variance, standard deviation, moment generating function; Properties and operations of expectations and variances. Continuous Random Variables: Definition and examples of continuous random variables; Probability density function and cumulative distribution function; Uniform, exponential, gamma, normal distributions; Expectation, variance, standard deviation, moment generating function; Properties and operations of expectations and variances.

Recommended Textbooks:

1. A First Course in Probability by Sheldon Ross
2. Introduction to Probability by Joseph K. Blitzstein and Jessica Hwang
3. Probability Theory: The Logic of Science by E. T. Jaynes
4. Probability: For the Enthusiastic Beginner by David J. Morin

MATH-5105: Number Theory Credit Hrs. 3(3-0)

Course Description: This course introduces fundamental concepts in number theory, including properties of integers, divisibility, prime numbers, modular arithmetic, and basic applications.

Course Contents:

1. **Fundamental Properties of Integers**
    - Division algorithm and greatest common divisor (GCD)
    - Euclidean algorithm
    - Fundamental theorem of arithmetic
2. **Divisibility and Prime Numbers**
    - Divisibility rules
    - Prime numbers and composite numbers
    - Unique factorization theorem
    - Infinitude of prime numbers
3. **Congruences and Modular Arithmetic**
    - Congruence relations
    - Modular arithmetic and basic properties
    - Solving linear congruences
    - Chinese remainder theorem
4. **Fermat's Little Theorem and Euler's Totient Function**
    - Fermat's Little theorem and its applications
    - Euler's Totient function and its properties
5. **Primitive Roots and Discrete Logarithms**
    - Primitive roots modulo n
    - Discrete logarithm problem
    - Applications in cryptography
6. **Quadratic Residues and Quadratic Reciprocity**
    - Quadratic residues and non-residues
    - Law of quadratic reciprocity
    - Applications of quadratic reciprocity
7. **Diophantine Equations and Applications**
    - Linear Diophantine equations
    - Pythagorean triples

- Fermat's Last Theorem (overview)

8. **Number-Theoretic Functions**
    - Multiplicative functions
    - Euler's phi function and Mobius function
    - Dirichlet convolution

Recommended Textbooks:

[1] "Elementary Number Theory" by Kenneth H. Rosen
[2] “Elements of Number Theory” by John Stillwell
[3] “An Introduction to the Theory of Numbers” by G.H. Hardy and E.M. Wright
[4] “Introductory Algebraic Number Theory” by Cambridge University Press

Courses for Semester-VII

MAT-6301 Partial Differential Equations 3(3-0)

Prerequisite(s): Ordinary Differential Equations, Real analysis

Specific Objectives of the Course:

The course provides a foundation to solve Partial Differential Equations with special emphasis on wave, heat and Laplace equations. Formulation and some theory of these equations are also intended.

Course Outline:

First-order-partial-differential equations; classification of second-order PDE; canonical form for second-order equations; wave, heat and the Laplace equation in Cartesian, cylindrical and spherical-polar coördinates; solution of partial differential equation by the methods of: separation of variables; the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations.

Recommended Books:

[1] Myint UT, Partial Differential Equations for Scientists and Engineers (3rd edition), 1987, North Holland, Amsterdam.

[2] Sneddon lN, Elements of partial Differential Equation, 1987, McGraw-Hill Books Company.

[3] Dennemyer R, Introduction to Partial Differential Equations and Boundary Value Problems, 1968, McGraw-Hill Book Company.

[4] Humi M, Miller WB, Boundary Value Problems and Partial Differential Equations, 1992, PWS-Kent Publishing Company, Bostan.

[5] Chester CR, Techniques in Partial Differential Equations, 1971, McGraw-Hill Book Company.

MAT-6302 Fluid Mechanics-I 3(3-0)

Prerequisite(s): Classical Mechanics

Specific Objectives of the Course:

The course objective is to provide students with the fundamental physical and analytical principles of fluid mechanics through the understanding of the: conservation of mass, conservation of energy, and the conservation of momentum equations. The student will demonstrate the understanding of these fundamentals by solving problems dealing with: fluid properties, fluid statics, pressure on plane and curved surfaces, buoyancy and floatation, kinematics, systems, control volumes, conservation principles, ideal incompressible flow, impulse-momentum, and flow of a real fluid.

Course Outline:

Real fluids and ideal fluids, Velocity of a fluid at a point, Equation of continuity, Acceleration of a fluid conditions at a rigid boundary, general analysis of fluid motion, Streamlines and path lines, Steady and unsteady flows, Velocity potential, Vorticity vector, Local and particle rates of change, Boundary Conditions, Boundary surface and its applications. Euler's equations of motion, Bernoulli's equation Steady motion under conservative body forces, Some potential theorems. Sources sink and doublets, Images in rigid infinite plane and solid spheres, A-symmetric flows, Stokes's stream function. Stream function, Complex potential for two-dimensional, Irrotational, Incompressible flow, Complex velocity potential for uniform stream, Line Sources and line sinks, Line doublets and line vortices, Image system, Milne-Thomson circle theorem, Blasius's theorem, The use of conformal transformation and the Schwarz-Christoffel transformation in solving problems, Vortex rows, Kelvin's Minimum energy theorem, Uniqueness theorem, Fluids streaming past a circular cylinder, Irrotational motion produced by a vortex filament, Karman's vortex-street.

Recommended Books:

[1] Charlton F, Textbook of fluid Dynamics, 1967, D. Van Nostrand Co. Ltd.
[2] Thomson M, Theoretical Hydrodynamics, 1979, Macmillan Press.
[3] Jaunzemics W, Continuum Mechanics, 1967, Macmillan Company.
[4] Landu LD, Lifshitz EM, Fluid Mechanics, 1966, Pergamon Press.
[5] Batchelor GK, An Introduction to Fluids Dynamics, 1969, Cambridge University Press.

MAT-6303 Mathematical Statistics 3(3-0)

Prerequisite(s): Probability Theory

Specific Objectives of the Course:

In the course "Probability Theory" the students learnt how to set up mathematical models of processes and systems that are affected by *chance*. In the present course the students would learn how to check these models against reality, to determine whether they are reliable/accurate enough for practical purposes or otherwise. This helps in making predictions and decisions.

Course Outline:

Sampling theory: sampling distributions; sampling procedures; estimation of parameters: estimation of mean, variance; confidence intervals; decision theory: hypothesis testing and decision making; types of errors in tests; quality control; control charts for mean, standard deviation, variance, range; goodness of fit, chi-square test; regression analysis; method of least squares; correlation analysis.

Recommended Books:

[1] DeGroot MH, Schervish MJ, Probability and Statistics (3rd edition), 2002, Addison-Wesley, Reading, Ma, USA (suggested text).
[2] Johnson RA, Probability and Statistics for Engineers, 1994, Prentice-Hall, Englewood Cliffs, NJ, USA.
[3] Papoulis A, Probability, Random Variables, and Stochastic Processes, (3rd edition), 1991, McGraw Hill, New York.
[4] Sincich T, Statistics by Examples, 1990, Dellen Publication Company.

MAT-6304 Functional Analysis-II 3(3-0)

Prerequisite(s): Functional Analysis-I

Specific Objectives of the Course:

Upon successful completion of this course students will be able to familiar with the major concepts of modern functional analysis, which certainly increase their level of mathematical maturity, including discovering and writing their own proofs.

Course Outline:

The Hahn-Banach theorem, Principle of uniform boundedness, Open mapping theorem, Closed graph theorem, Weak topologies and the Banach-Alaoglu theorem, Extreme points and the Klien-Milman theorem, The dual and bidual spaces, Reflexive spaces, Compact operators, Spectrum and eigenvalues of an operator, Elementary spectral theory.

Recommended Books:

[1] Kreyszing E, Introductory Functional Analysis and Applications, 1973, John Wiley.
[2] Taylor E, Lay DC, Introduction of Functional Analysis, 1979, John Wiley.
[3] Heuser HG, Functional Analysis, 1982, John Wiley.
[4] Groetsch CW, Elements of Applicable of Applicable Functional Analysis, 1980, Marcel Dekker.

MAT-6305 **Measure Theory** **3(3-0)**

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

To introduce the concepts of *measure* and *integral with respect to a measure*, to show their basic properties, and to provide a basis for further studies in Analysis, Probability, and Dynamical Systems. Moreover, to gain understanding of the abstract measure theory and definition and main

properties of the integral. To construct Lebesgue's measure on the real line and in n-dimensional Euclidean space. To explain the basic advanced directions of the theory.

Course Outline:

Measure Space; Definition and examples of algebra and σ-algebra, Basic properties of measurable spaces, Definition and examples of measure spaces, Outer measure, Lebesgue measure, Measurable sets, complete measure spaces.

Measurable Functions; Some equivalent formulations of measurable functions, Examples of measurable functions, Various characterization of measurable functions, Properties that holds almost everywhere.

Lebesgue Integration; Definition of Lebesgue integral, Basic properties of Lebesgue integrals, Comparison between Riemann integration and Lebesgue integration, L_2-Spaces, The Riesz Fischer theorem.

Recommended Books:

[1] Royden HL, Real Analysis, 1968, Mecmillian.
[2] Cohn DL, Measure Theory, 980, Birkhauser.
[3] Halmos PR, Measure Theory, 1950, D. Van Nostrand.
[4] Khan AR, Introduction to Lebesgue Integration, 2003, Ilmi Kitab Khanna, Urdu Bazar, Lahore.

MAT-6306 **Modeling and Simulation** **3(2-1)**

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

Course Outline:

Concepts of model, modeling and simulation, functions, linear equations, linear-differential equations, nonlinear-differential equations and integral equations as models, introduction to simulation techniques.

Ordinary-Differential Equations: Modeling with first order differential equations: Newton's law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; application to biological systems; modeling with periodic or impulse forcing functions. Modeling with systems of first order differential equations; competitive hunter model; predator-prey model.

Partial-Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding. Modeling wave phenomena (wave equation); shallow water waves, uniform transmission line, traffic flow, RC circuits. Modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc. Modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems. Equation of continuity.

Simulation: Techniques of simulation (students are required to simulate at least one system).

Recommended Books:

[1] Giordano FR, Weir MD, Differential Equations: A Modeling Approach, 1994, Addison-Wesley, Reading, Ma, USA (suggested text).
[2] Jerri AJ, Introduction to Integral Equations with Applications, 1985, Marcel Dekker, New York.
[3] Myint UT, Debnath L, Partial Differential Equations for Scientists and Engineers (3rd edition), 1987, North Holland, Amsterdam.

MAT-6307 Optimization Theory 3(3-0)

Prerequisite(s): Real Analysis-I

Specific Objectives of the Course:

The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

Course Outline:

Linear programming: simplex method, duality theory, dual and primal-dual simplex methods. Unconstrained optimization: optimality conditions, one-dimensional problems, multi-dimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem. The calculus of variations, the Euler-Lagrange equations, functionals depending on several variables, variational problems in parametric form, transportation models and networks.

Recommended Books:

[1] Elsgolts L, Differential Equations and the Calculus of Variations, 1970, Mir Publishers, Moscow.
[2] Gotfried BS, Weisman J, Introduction to Optimization Theory, 1973, Prentice Hall, Englewood Cliffs, NJ, USA.
[3] Luenberger DG, Introduction to Linear and Non-Linear Programming, 1973, Addision-Wesley, Reading, Ma, USA.

MAT-6308 Numerical Methods for Partial Differential Equations 3(3-0)

Prerequisite(s): Numerical Analysis-I

Specific Objectives of the Course:

Many application problems lead to partial differential equations in which analytic solutions are rarely available or too complicated. Finite difference methods are often easy to use but powerful to obtain an approximate solution of the PDE. It is strongly believed that the knowledge of *Finite*

Difference Methods for PDEs is important for mathematicians, scientists, and engineers, who are interested in solving their problems approximately. It is a required course in many universities. This course is designed for students in applied mathematics, engineering, and the sciences to learn the basic *theories and algorithms* of finite difference methods for differential equations including elliptic, parabolic and hyperbolic PDE's. While theoretical foundations will be described, emphasis will also be placed on algorithm design and implementation. We will also explore available software packages in this field.

Course Outline:

Introduction (ODE/PDE and classification, analytic approaches versus numerical approximation, finite difference versus finite element method); A model problem (two point boundary value problem) and the finite difference method; Finite difference methods basics, stability and consistency, etc; Finite difference method for general one dimensional elliptic boundary value problems with different boundary conditions; Finite difference methods for two dimensional elliptic PDEs (may include multi-grid and fast Poisson solvers); Finite difference methods for one and two dimensional parabolic PDEs, e.g. the heat equation; von Neumann stability analysis and Fourier transforms, ADI method; Finite difference methods for one and two dimensional hyperbolic PDEs, e.g. the wave equation, numerical methods for conservation laws; Advanced topics, irregular domain, the level set method etc. if time permits.

Recommended Books:

[1] LeVeque RJ, Finite Difference Methods for Ordinary and Partial Differential Equations, 2007, SIAM.

[2] Strilwerda JC, Finite Difference Schemes and Partial Differential Equations, 1989, Chapman & Hall.

[3] Morton KW, Mayers DF, Numerical Solution of Partial Differential Equations, 1995, Cambridge press.

MAT-6309 Theory of Differential Equations 3(3-0)

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

The aim of this course is to introduce the students to the basic theory of ordinary differential equations and give a competence in solving ordinary differential equations by using analytical or numerical methods. The subject of differential equations is a very important branch of applied mathematics. Many phenomena from physics, biology and engineering may be described using ordinary differential equations. In order to understand the underlying processes we have to find and interpretate the solutions of these equations. This course explains different methods of solution of ordinary differential equations: from analytical to numerical.

Course Outline:

First-Order Differential Equations; Linear Systems; Autonomous Systems; Perturbation Methods; The Self-Adjoint Second-Order Differential Equation; Linear Differential Equations of Order n; BVPs for Nonlinear Second-Order DEs; Existence and Uniqueness Theorems.

Recommended Books:

[1] Hartman P, Ordinary Differential Equations, 1964, John Wiley, New York.

[2] Hubbard JH, West BH, Differential Equations: a Dynamical Systems Approach, Higher-Dimensional Systems, 1995, Springer-Verlag, New York.

[3] Iooss G, Joseph DD, Elementary Stability and Bifurcation Theory, Undergraduate Texts in Mathematics, 1980, Springer-Verlag, New York.

[4] Walter GK, Allan CP, The Theory of Differential Equations: Classical & Qualitative, 2010, Springer, New York.

MAT-6310 Analysis and Control of Nonlinear Systems 3(3-0)

Prerequisite(s): Ordinary Differential Equations

Specific Objectives of the Course:

The course aims to provide an overview of techniques for analysis and control design for nonlinear systems. Whereas linear system control theory is largely based on linear algebra (e.g. when determining the behavior of solutions to linear differential equations) and complex analysis (e.g. when predicting system behavior from transfer functions), the broader range of behavior exhibited by nonlinear systems requires a wider variety of techniques. The course gives an introduction to some of the most useful and commonly used tools for determining system behavior from a description in terms of differential equations.

Course Outline:

Introduction to Nonlinear Phenomena: Multiple Equilibria, Limit Cycles, Complex Dynamics, Bifurcations. Second Order Nonlinear Systems: Phase Plane Techniques, Limit Cycles - Poincare-Bendixson Theory, Index Theory. Input-output analysis and stability: Small Gain Theorem, Passivity, Describing Functions. Lyapunov Stability Theory: Basic stability and instability theorems, LaSalle's theorem, Indirect method of Lyapunov. Linearization by State Feedback: Input-Output and Full State Linearization, Zero Dynamics, Inversion, Tracking, Stabilization.

Recommended Books:

[1] Sastry SS, Nonlinear Systems: Analysis, Stability, and Control, 1999, Springer-Verlag.
[2] Khalil HK, Nonlinear Systems, 2002, Prentice-Hall.

MAT-6311 Computing Tools for Mathematicians 3(2-1)

Prerequisite(s): Programming Languages for Mathematicians

Specific Objectives of the Course:

The purpose of this course is to teach students the use of mathematical software like MATLAB, MAPLE, MATHEMATICA for solving computationally-difficult problems in mathematics. The student shall become well versed in using at least one of the mathematical software and shall learn a number of techniques that are useful in calculus as well as in other areas of mathematics.

Course Outline:

The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above.

Recommended Books:

[1] Etter DM, Kuncicky D, Hull D, *Introduction to MATLAB 6*, 2001, Prentice Hall, Englewood Cliffs, NJ, USA.
[2] Garvan F, The Maple Book, 2002, Chapman & Hall/CRC.
[3] Kaufmann S, Mathematica as a Tool: An Introduction with Practical Examples, 1994, Springer, New York.

MAT-6312 Vector and Tensor Analysis 3(3-0)

Prerequisite(s): Calculus- II

Specific Objectives of the Course:

This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics.

Course Outline:

3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coordinate systems and their bases, the spherical-polar- and the cylindrical-coordinate meshes, tensors of first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagnolization of inertia tensor as aligning coordinate frame with natural symmetries of the system.

Recommended Books:

[1] Bourne DE, Kendall PC, Vector Analysis and Cartesian Tensors (2^{nd} edition), Thomas Nelson.
[2] Shah NA, Vector and Tensor Analysis, 2005, A-One Publishers, Lahore.
[3] Smith GD, Vector Analysis, Oxford University Press, Oxford.
[4] Spiegel MR, Vector Analysis, 1974, McGraw Hill, New York.